RÉGLAGE DU RFQ D' IPHI:

CELLULES TERMINALES ET CELLULES DE COUPLAGE

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CELLULES TERMINALES ET CELLULES DE COUPLAGE

1 MODÈLE ÉLECTRIQUE

LIGNE QUADRIFILAIRE CHARGÉE

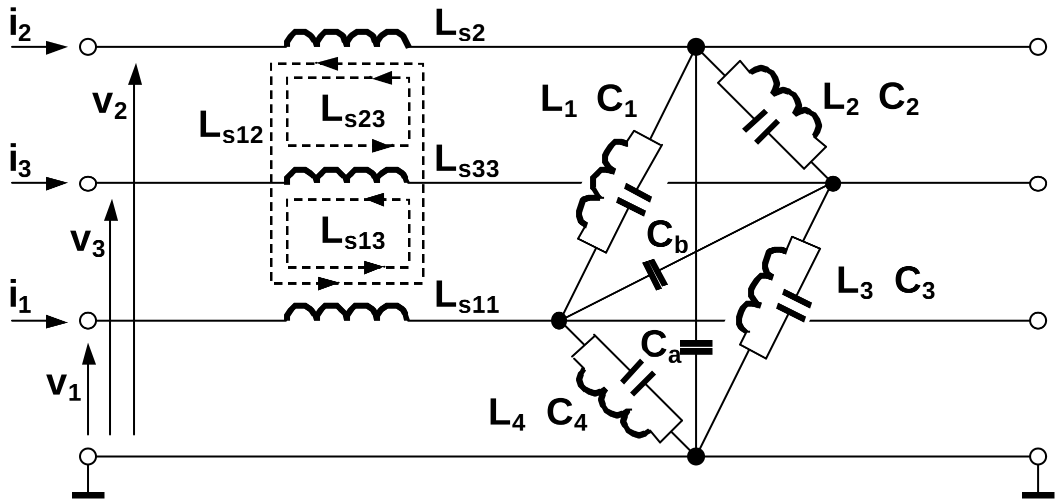
CELLULES TERMINALES: CIRCUITS À 3 PORTS

CELLULES DE COUPLAGE: CIRCUITS À 6 PORTS

2 RÉSULTATS EXPÉRIMENTAUX

LOADED 4-WIRE LINE

EQUIVALENT TO RFQ AXIAL REGION UP TO MICRO-WAVE FREQUENCIES



$$\frac{\partial v}{\partial z} = -\, j\omega\, L_s\, i$$

$$\frac{\partial i}{\partial z} = -\left(j\omega\, C + \frac{1}{j\omega} L \right) v$$

$$\downarrow$$

$$-\frac{\partial}{\partial z}\left(C\,\frac{\partial v}{\partial z}\right) + \frac{1}{c^2} L\, v = \frac{\omega^2}{c^2} C\, v$$

$$L := \begin{vmatrix} L_1^{-1} + L_4^{-1} & -L_1^{-1} & 0 \\ -L_1^{-1} & L_2^{-1} + L_1^{-1} & -L_2^{-1} \\ 0 & -L_2^{-1} & L_3^{-1} + L_2^{-1} \end{vmatrix}$$

uniform approximation:

$$-\frac{\partial^2 v}{\partial z^2} + \frac{1}{c^2} C^{-1} L\, v = \frac{\omega^2}{c^2} v$$

$$C := \begin{vmatrix} C_1 + C_4 + C_b & -C_1 & -C_b \\ -C_1 & C_2 + C_1 + C_a & -C_2 \\ -C_b & -C_2 & C_3 + C_2 + C_b \end{vmatrix}$$

$$\nu^2\, L_S\, C = I_d$$

LOADED 4-WIRE LINE EQUATION: *VOLTAGE BASIS*

REFERENCED VOLTAGES

$$\begin{vmatrix} v_1 \\ v_2 \\ v_3 \end{vmatrix} = \begin{vmatrix} -1 & 0 & -1 \\ 0 & +1 & -1 \\ -1 & +1 & 0 \end{vmatrix} \begin{vmatrix} U_Q \\ U_S \\ U_T \end{vmatrix}$$

$$v = S\,U$$

INTER-ELECTRODE VOLTAGES

$$\begin{vmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{vmatrix} = \begin{vmatrix} +1 & +1 & 0 \\ -1 & 0 & +1 \\ +1 & -1 & 0 \\ -1 & 0 & -1 \end{vmatrix} \begin{vmatrix} U_Q \\ U_S \\ U_T \end{vmatrix}$$

4 – WIRE LINE EQUATION **:** $-\dfrac{\partial^2 U}{\partial z^2} + A\,U = \dfrac{\omega^2}{c^2}\,U$

perfectly (quaternary) symmetrical case **:**

$$A = \frac{1}{c^2} S\,C^{-1}\,L\,S^{-1} = \begin{vmatrix} \dfrac{1}{c^2 L_0 C_0} & 0 & 0 \\ 0 & \dfrac{1}{c^2 L_0 (C + C_a)} & 0 \\ 0 & 0 & \dfrac{1}{c^2 L_0 (C_0 + C_a)} \end{vmatrix}$$

LOADED 4-WIRE LINE: *CURRENT BASIS*

$$\begin{vmatrix} i_1 \\ i_2 \\ i_3 \end{vmatrix} = \begin{vmatrix} -1/2 & -1/2 & -1/2 \\ +1/2 & +1/2 & -1/2 \\ -1/2 & +1/2 & +1/2 \end{vmatrix} \begin{vmatrix} I_Q \\ I_S \\ I_T \end{vmatrix}$$

$$\begin{vmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{vmatrix} = \begin{vmatrix} -1/2 & -1/2 & -1/2 \\ +1/2 & +1/2 & -1/2 \\ -1/2 & +1/2 & +1/2 \\ +1/2 & -1/2 & +1/2 \end{vmatrix} \begin{vmatrix} I_Q \\ I_S \\ I_T \end{vmatrix}$$

$$i = W^{-1} I$$

LINE EQUATIONS IN NEW BASIS:

$$\begin{vmatrix} I = -\frac{1}{j\omega} c^2 C_Q \frac{\partial U}{\partial z} \\ \frac{\partial I}{\partial z} = -\left(j\omega C_Q + \frac{1}{j\omega} L_Q \right) U \end{vmatrix}$$

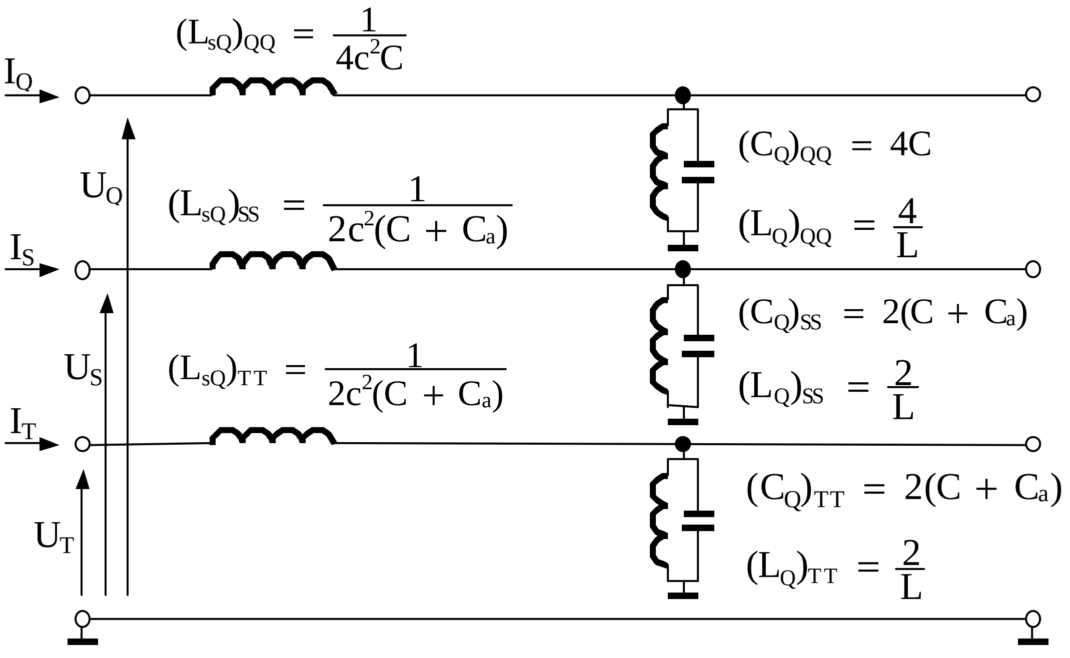
capacitance (symmetrical) matrix in new basis:

$$C_Q := W C S^{-1} = \begin{vmatrix} +C_1 + C_2 + C_3 + C_4 & +C_1 - C_3 & -C_2 + C_4 \\ +C_1 - C_3 & +C_1 + C_3 + C_a + C_b & -C_a + C_b \\ -C_2 + C_4 & -C_a + C_b & +C_1 + C_3 + C_a + C_b \end{vmatrix}$$

inverse-inductance (symmetrical) matrix in new basis:

$$L_Q := W L S^{-1} = \begin{vmatrix} +L_1^{-1} + L_2^{-1} + L_3^{-1} + L_4^{-1} & +L_1^{-1} - L_3^{-1} & -L_2^{-1} + L_4^{-1} \\ +L_1^{-1} - L_3^{-1} & +L_1^{-1} + L_3^{-1} & 0 \\ -L_2^{-1} + L_4^{-1} & 0 & +L_1^{-1} + L_3^{-1} \end{vmatrix}$$

LOADED 4-WIRE LINE *EQUATIONS IN {Q,S,T} BASIS*



perfectly (quaternary) symmetrical case:
all matrixes are diagonal

$$\frac{\partial U}{\partial z} = -j\omega L_{sQ}\, I$$

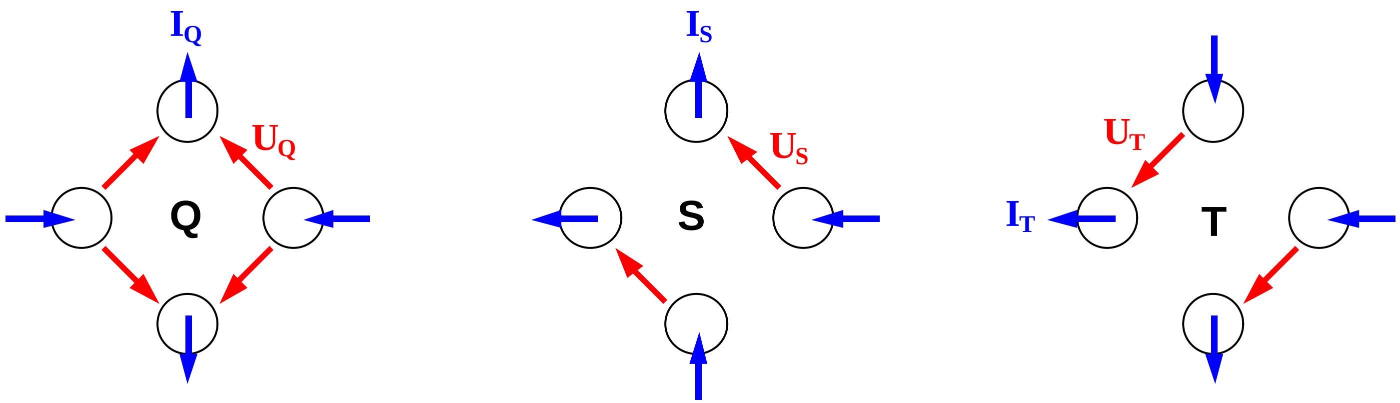
$$\frac{\partial I}{\partial z} = -\left(j\omega C_Q + \frac{1}{j\omega} L_Q \right) U$$

$$\downarrow$$

$$-\frac{\partial^2 U}{\partial z^2} + A\, U = \frac{\omega^2}{c^2} U$$

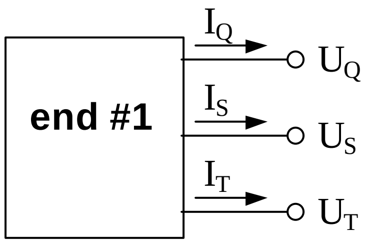
$$A = \frac{1}{c^2} C_Q^{-1} L_Q$$

LOADED 4-WIRE LINE: *VOLTAGE AND CURRENT BASES*



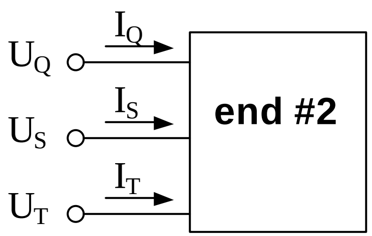
END-CELLS MODEL

END-CELLS ARE GENERAL 3-PORT CIRCUITS



$$I(a) = -Y_a \, U(a)$$

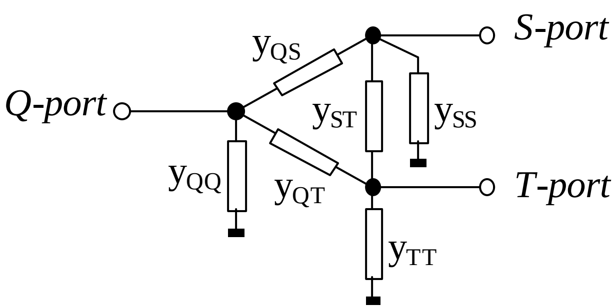
$$I(b) = +Y_b \, U(b)$$

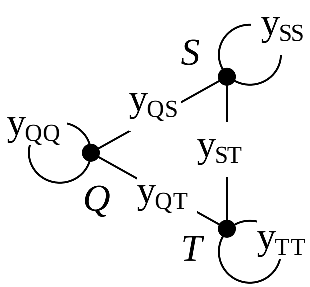


$$Y = \begin{vmatrix} y_{QQ} + y_{SQ} + y_{TQ} & -y_{QS} & -y_{QT} \\ -y_{QS} & y_{SS} + y_{QS} + y_{ST} & -y_{ST} \\ -y_{QT} & -y_{ST} & y_{TT} + y_{QT} + y_{ST} \end{vmatrix}$$

reciprocity = matrix symmetry

lossless = pure imaginary





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END-CELLS MODEL

END-CELLS DEFINE BOUNDARY CONDITIONS

$$\begin{vmatrix} \partial_Z U_Q(a) \\ \partial_Z U_S(a) \\ \partial_Z U_T(a) \end{vmatrix} := - \begin{vmatrix} (s_a)_{QQ} & (s_a)_{QS} & (s_a)_{QT} \\ (s_a)_{QS} & (s_a)_{SS} & (s_a)_{ST} \\ (s_a)_{QT} & (s_a)_{ST} & (s_a)_{TT} \end{vmatrix} \begin{vmatrix} U_Q(a) \\ U_S(a) \\ U_T(a) \end{vmatrix}$$

Tuned RFQ:

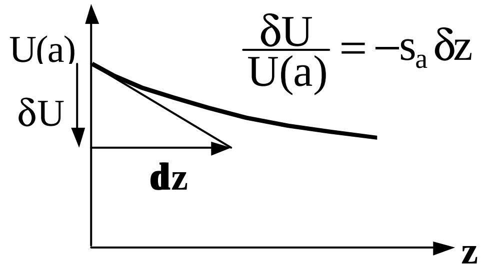
$$U_S(a) = U_T(a) = 0$$

$$\partial_z U(b) := +s_b\, U(b)$$

ADMITANCE AND LOGARITHMIC-SLOPE MATRIXES ARE RELATED VIA LINE EQUATION:

$$\frac{\partial U}{\partial z} = -j\omega\, L_{sQ}\, I \longrightarrow s_{a,b} = -j\omega\, L_{sQ}\, Y_{a,b}$$

***INDUCTIVE CELL* s < 0 ; *CAPACITIVE CELL* s > 0**

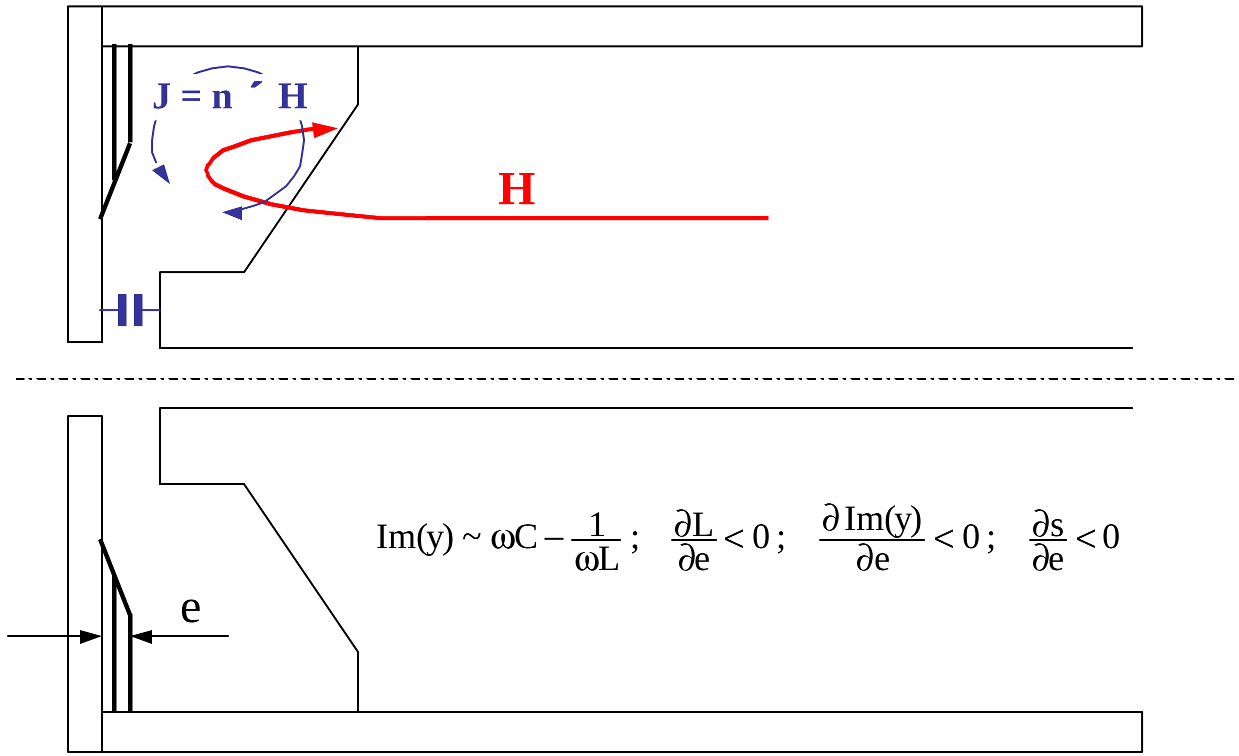


$$\frac{\delta U}{U(a)} = -s_a\, \delta z$$

's' *UNITS:* (V/m) / V

END-CELLS MECHANICAL TUNING

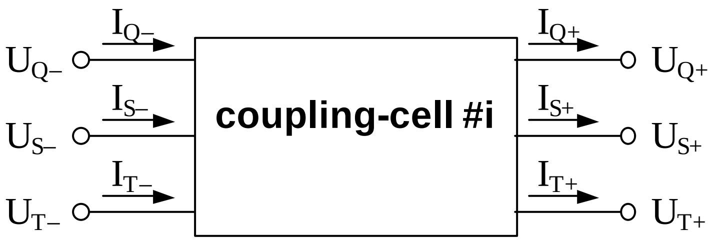
INDUCTIVE TUNING BY ADJUSTMENT OF PLATE THICKNESS

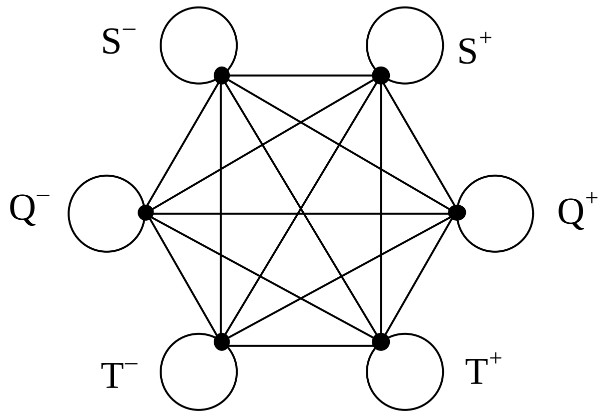


$$\mathrm{Im}(y) \sim \omega C - \frac{1}{\omega L}\ ; \quad \frac{\partial L}{\partial e} < 0\ ; \quad \frac{\partial\, \mathrm{Im}(y)}{\partial e} < 0\ ; \quad \frac{\partial s}{\partial e} < 0$$

COUPLING-CELLS MODEL

COUPLING-CELLS ARE GENERAL 6-PORT CIRCUITS





$$\begin{vmatrix} I^{-}(z_i) \\ -\ I^{+}(z_i) \end{vmatrix} = Y_i \begin{vmatrix} U^{-}(z_i) \\ U^{+}(z_i) \end{vmatrix}$$

$$Y_{XY} = -y_{XY}\ , \quad X \neq Y\ , \quad X\ , Y \in \left\{Q^{-}, S^{-}, T^{-}, Q^{+}, S^{+}, T^{+}\right\}$$

$$Y_{XX} = y_{XX} - \sum_{Y \neq X} y_{XY}\ , \quad X\ , Y \in \left\{Q^{-}, S^{-}, T^{-}, Q^{+}, S^{+}, T^{+}\right\}$$

COUPLING-CELLS MODEL

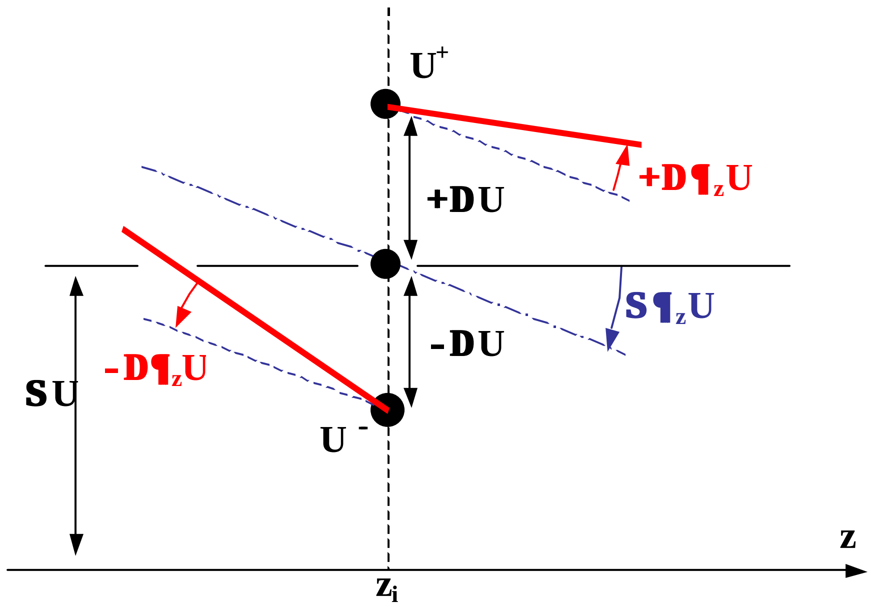
COUPLING-CELLS DEFINE BOUNDARY CONDITIONS

$$
\begin{vmatrix}
\partial_Z U_Q^-(z_i) \\
\partial_Z U_S^-(z_i) \\
\partial_Z U_T^-(z_i) \\
-\partial_Z U_Q^+(z_i) \\
-\partial_Z U_S^+(z_i) \\
-\partial_Z U_T^+(z_i)
\end{vmatrix}
:=
\begin{vmatrix}
(s_i)_{Q^-Q^-} & (s_i)_{Q^-S^-} & (s_a)_{Q^-T^-} & (s_i)_{Q^-Q^+} & (s_i)_{Q^-S^+} & (s_i)_{Q^-T^+} \\
(s_i)_{Q^-S^-} & (s_i)_{S^-S^-} & (s_i)_{S^-T^-} & (s_i)_{S^-Q^+} & (s_i)_{S^-S^+} & (s_i)_{S^-T^+} \\
(s_i)_{Q^-T^-} & (s_a)_{S^-T^-} & (s_i)_{T^-T^-} & (s_i)_{T^-Q^+} & (s_i)_{T^-S^+} & (s_i)_{T^-T^+} \\
(s_i)_{Q^-Q^+} & (s_i)_{S^-Q^+} & (s_i)_{T^-Q^+} & (s_i)_{Q^+Q^+} & (s_i)_{Q^+S^+} & (s_a)_{Q^+T^+} \\
(s_i)_{Q^-S^+} & (s_i)_{S^-S^+} & (s_i)_{T^-S^+} & (s_a)_{Q^+S^+} & (s_i)_{S^+S^+} & (s_i)_{S^+T^+} \\
(s_i)_{Q^-T^+} & (s_i)_{S^-T^+} & (s_i)_{T^-T^+} & (s_i)_{Q^+T^+} & (s_i)_{S^+T^+} & (s_i)_{T^+T^+}
\end{vmatrix}
\begin{vmatrix}
U_Q^-(z_i) \\
U_S^-(z_i) \\
U_T^-(z_i) \\
U_Q^+(z_i) \\
U_S^+(z_i) \\
U_T^+(z_i)
\end{vmatrix}
$$

ADMITANCE LOGARITHMIC-SLOPE MATRIXES ARE RELATED VIA LINE EQUATION:

$$
\frac{\partial U}{\partial z} = -j\omega L_{sQ} I \longrightarrow s_i = -j\omega \begin{vmatrix} L_{sQ} & 0 \\ 0 & L_{sQ} \end{vmatrix} Y_i
$$

COUPLING-CELLS TUNING



ODD AND EVEN QUANTITIES

$$U^- := \Sigma U - \Delta U \qquad \partial_z U^- := \Sigma\partial_z U - \Delta\partial_z U$$

$$U^- := \Sigma U + \Delta U \qquad \partial_z U^- := \Sigma\partial_z U + \Delta\partial_z U$$

OPEN-CIRCUIT AND COUPLING SLOPES

$$\begin{vmatrix}\partial_z U^- \\ -\partial_z U^+\end{vmatrix} = \begin{vmatrix} s_o^- + s_c & -s_c \\ -s_c & s_o^+ + s_c\end{vmatrix}\begin{vmatrix}U^- \\ U^+\end{vmatrix}$$

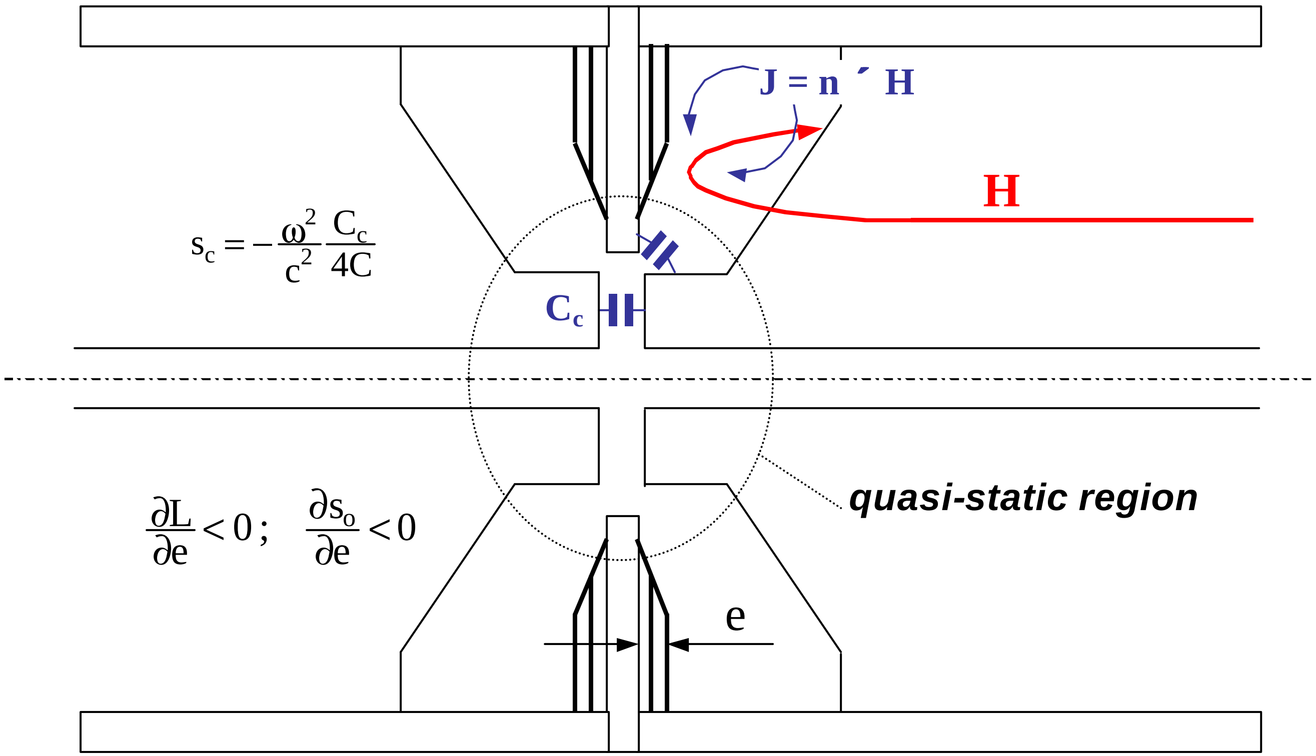
MEAN VOLTAGE SLOPE ACROSS CELL

$$\begin{vmatrix}\Sigma\partial_z U \\ \Delta\partial_z U\end{vmatrix} = \begin{vmatrix} \frac{s_o^- - s_o^+}{2} & -\frac{s_o^- + s_o^+}{2} - 2s_c \\ -\frac{s_o^- + s_o^+}{2} & \frac{s_o^- - s_o^+}{2}\end{vmatrix}\begin{vmatrix}\Sigma U \\ \Delta U\end{vmatrix}$$

COUPLING-CELL BALANCE

COUPLING-CELLS MECHANICAL TUNING

INDUCTIVE TUNING BY ADJUSTMENT OF PLATE THICKNESS



ESTIMATION OF END-CELL MATRIX

M **KNOWN COUPLES OF (MEASURED) VECTORS OF** $\mathbf{R}^3$ **:** $\left\{ U_m(a) \ , \ \partial_z U_m(a) \right\}_{1 \le m \le M}$

ARE SUPPOSED TO VERIFY : $\partial_z U_m(a) = -s_a \, U_m(a)$

THEN s_a **MAY BE ESTIMATED IN A LEAST-SQUARE SENSE IF AND ONLY IF:**

$$M \geq 3$$

$$\text{rank} \ \{ U_m(a) \}_{1 \le m \le M} = 3$$

ESTIMATION OF COUPLING-CELL MATRIX

M **KNOWN COUPLES OF (MEASURED) VECTORS OF** $\mathbf{R}^6$ **:**

$$\left\{ \begin{vmatrix} U_m^-(z_i) \\ U_m^+(z_i) \end{vmatrix} , \begin{vmatrix} \partial_z^- U_m(z_i) \\ \partial_z^+ U_m(z_i) \end{vmatrix} \right\}_{1 \le m \le M}$$

ARE SUPPOSED TO VERIFY :

$$\begin{vmatrix} \partial_z^- U_m(z_i) \\ -\partial_z^+ U_m(z_i) \end{vmatrix} = s_i \begin{vmatrix} U_m^-(z_i) \\ U_m^+(z_i) \end{vmatrix}$$

THEN s_i **MAY BE ESTIMATED IN A LEAST-SQUARE SENSE IF AND ONLY IF:**

$$M \geq 6$$

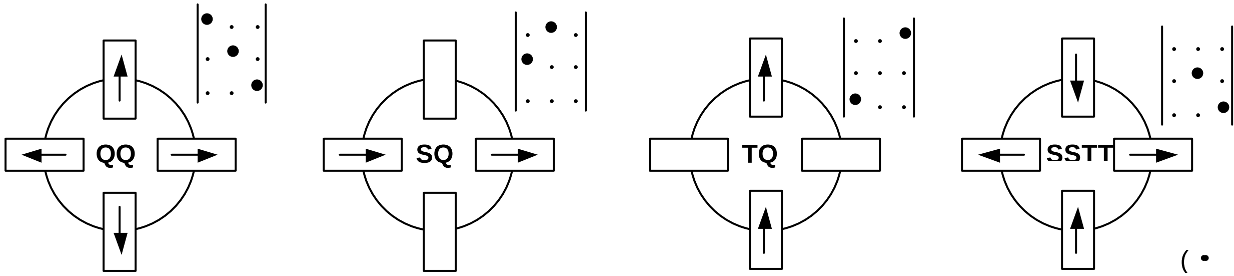
$$\operatorname{rank} \left\{ \begin{vmatrix} U_m^-(z_i) \\ U_m^+(z_i) \end{vmatrix} \right\}_{1 \le m \le M} = 6$$

SETS OF INDEPENDANT EXCITATIONS

IDEA: • **USE INFINITESIMAL GENERATORS OF** $-\frac{\partial^2}{\partial z^2} + A$ **:**

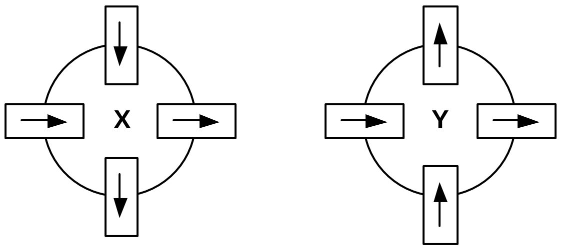
$$dA = G_{QQ} dL_{QQ} + G_{SQ} dL_{SQ} + G_{TQ} dL_{TQ} + G_{SSTT} dL_{SSTT} + \cdots$$

TO DERIVE INDEPENDANT EXCITATIONS OF END/COUPLING CELLS:



(• NON-ZERO ELEMENTS)

• **USE ADDITIONAL LINEAR COMBINATIONS TO POPULATE THE SET:**



ETC.

PRECISION AND ACCURACY

1 - *ABILITY OF THE METHOD TO MATCH THE DATA:*

COMPUTE BOUNDS ON THE COMPONENTS OF $\frac{\tilde{s}\, U_m - \partial_z U_m}{\| U_m \|}$

OVER THE FULL MEASUREMENT SET

2 - *STANDARD DEVIATION OF THE ESTIMATE:*

COMPUTE THE STANDARD DEVIATION OF M **SUCCESSIVE ESTIMATES OF** s **,**
USING ALL POSSIBLE SUB-SETS OF $M-1$ **EXCITATIONS**

3 - *ACCURACY:*

TEST ESTIMATES AGAINST VARIATION OF MECHANICAL TUNING PARAMETERS

SUMMARY OF EXPERIMENTS USING CEA / SACLAY COLD MODEL

{ QQ SQ TQ X Y }, 2.5 mm steps

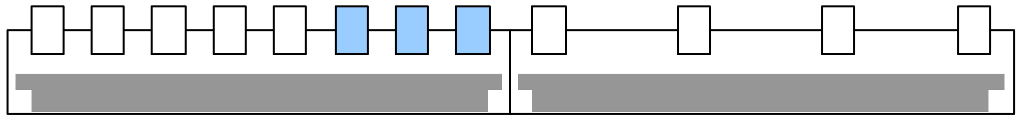
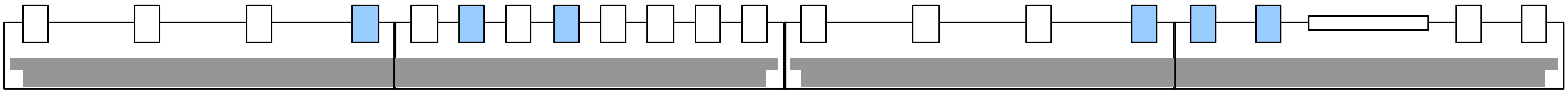


plate thickness =	0 / 10 / 20 mm	9 mm	9 mm	9 mm
dipole rod length =	148 mm	158.5 mm	158.5 mm	148 mm
plate thickness =	10 mm	9 mm	9 mm	9 mm
dipole rod length =	0 mm	158.5 mm	158.5 mm	148 mm

4 ´ 5 = 20 BEAD-PULLS

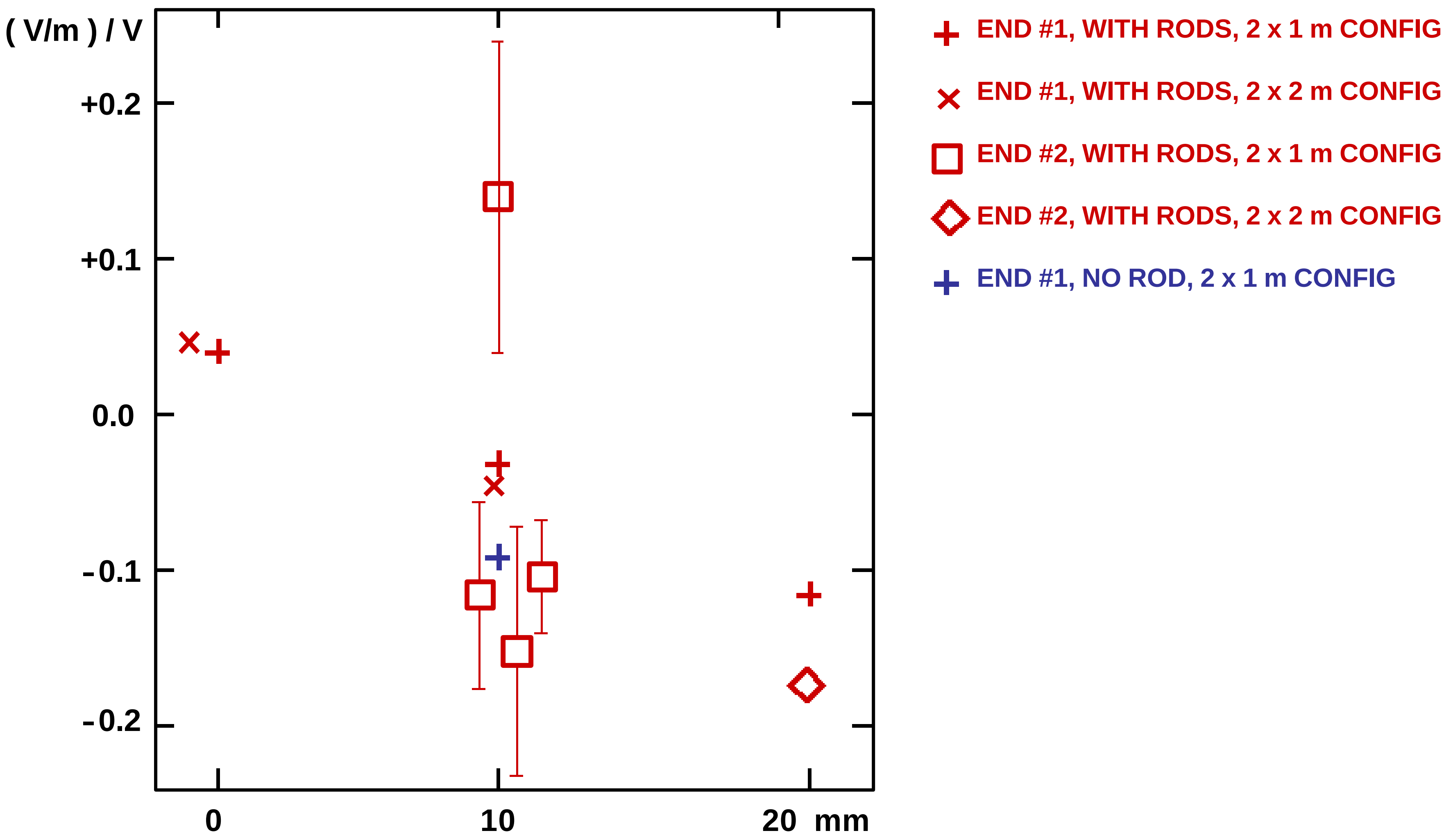
{ QQ SQ TQ SSTT X Y } **{ QQ SQ TQ SSTT X Y }**



10 mm 152 mm	2.5 mm steps	18 mm 160 mm	18 mm 160 mm	2.5 mm steps	10 mm 152 mm
0 mm 152 mm	7.5 mm steps	9 mm 160 mm	18 mm 160 mm	7.5 mm steps	20 mm 152 mm

2 ´ 11 = 22 BEAD-PULLS

END-CELL SUMMARY



END-CELL PRECISION ANALYSIS

1 - DATA MATCH: ± 0.01 TO ± 0.02 (V/m)/V TYPICAL , NEVER WORSE THAN ± 0.032 (V/m)/V

- **i.e. ± 0.4 % OVER THE DISTANCE BETWEEN END-PLATE AND CLOSEST FIELD POINT**
- **NOT CLEARLY RELATED TO NUMBER OF EXCITATIONS IN DATA SET, NOR TO TUNER STEP VALUE**

2 - STANDARD DEVIATION OF ESTIMATE:

0.05 TO 0.15 (V/m)/V WITH 2.5 mm TUNER STEPS

0.001 TO 0.003 (V/m)/V WITH 7.5 mm TUNER STEPS

- **STEPPED TUNERS AND END-CELL SHOULD BELONG TO THE SAME SEGMENT**
- **NOT CLEARLY RELATED TO NUMBER OF EXCITATIONS, AS SOON AS QQ, SQ , TQ , X AND Y ARE IN THE SET**

COUPLING-CELL SUMMARY

tuner step 7.5 mm	s_{Q-Q-}	+0.991	0.0526	(V/m) / V
	s_{Q+Q+}	+1.094	0.0094	(V/m) / V
	s_{Q-Q+}	−0.9936	0.0359	(V/m) / V
plate thickness				
9 mm	Q^- load	−0.0026	0.056	(V/m) / V
18 mm	Q^+ load	+0.100	0.039	(V/m) / V
	Q-coupling cap	2.21	0.08	pF

- STD DEV AS HIGH AS 0.15 (V/m)/V WITH 2.5 mm TUNER STEP
- DATA MATCH BETTER THAN ± 0.05 (V/m) / V WITH 7.5 mm TUNER STEP
- COUPLING-CAPACITANCE CLOSE TO ELECTROSTATIC THEORY (2.18 pF), IN GOOD AGREEMENT WITH EIGEN-FREQUENCIES
- RELATIVE VALUES OF Q^+ Q^- LOADS ARE PUZZLING, CAPACITIVE EFFECT MAY OVERCOME INDUCTIVE EFFECT
- COUPLING CAP RATIO: LOW S/Q = 1.876, CORRECT T/Q = 2.009, CORRELATES WITH ELECTRODE GAP DISPERSION (1.10 TO 1.57 mm)